Singular real plane sextic curves without real points

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It is a common understanding that any reasonable geometric question about $K3$-surfaces can be restated and solved in purely arithmetical terms, by means of an appropriately defined *homological type*. For example, this works well in the study of *singular complex* sextic curves or quartic surfaces (see [1, 2]), as well as in that of *smooth real* ones (see [6, 4]). However, when the two are combined (singular real curves or surfaces), the approach fails as the "obvious" concept of homological type does not fully reflect the geometry (*cf.*, *e.g.*, [3] or [5]).

We show that the situation can be repaired if the curves in question have empty real part or, more generally, have no *real* singular points; then, one can indeed confine oneself to the homological types consisting of the exceptional divisors, polarization, and real structure. Still, the resulting arithmetical problem is not quite straightforward, but we manage to solve it in the case of empty real part.

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References

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